A SPICE-based Emulator Framework for Quantum Error Correction Circuits using LC Resonators

Abstract

Quantum error correction (QEC) is vital for protecting quantum information against errors induced by quantum noise and decoherence, enabling fault-tolerant quantum computation. Here, we develop a SPICE-based classical emulator framework for simulating gate-based quantum error correction circuits for correcting bit-flip error and phase flip error. Here, we construct a SPICE-based classical emulator framework to simulate gate-based quantum error correction circuits, specifically designed to correct both bit-flip and phase flip errors. In our framework, a collection of LC-oscillators emulates the process behavior quantum gates. Each quantum state is perfectly described by the phase and amplitude of each oscillator. Our framework is scalable to any quantum error correction system with any arbitrary number of qubits since each gate process is perfectly achieved. Here, we have successfully conducted simulations for an individual bit-flip correction circuit and Shor's nine-bit error correcting quantum circuit, demonstrating the capability of our framework to effectively address and repair random bit-flip and phase flip errors in gate-based quantum computing circuits.

Keywords

Quantum circuit, LC oscillator, Classical emulator, Quantum computing, Universal gates.

1. Introduction

With classical computing reaching its boundaries, scientists are actively exploring alternative computational paradigms, and quantum computation emerges as a particularly successful contender [1], [2]. Quantum computers exhibit superior efficiency, demonstrated by applications like Shor's factorization algorithm and Grover's search algorithm, showcasing their potential advantages over classical counterparts [3]–[5]. The key to this efficiency lies in the structure of Hilbert space, where quantum states represent linear superpositions of classical binary states over a scalar field. As the number of qubits increases, Hilbert space expands exponentially, providing quantum computers with significantly enhanced computational capabilities [6]. This quantum advantage is exemplified by their ability to solve certain problems, such as quantum material simulation [7], millions of times faster than classical computers. However, Quantum computers are inherently susceptible to errors due to various factors such as environmental noise, and imperfect operations on qubits [8]–[10]. Unlike classical bits, which are binary and deterministic, qubits exist in superpositions of states, making them more fragile and susceptible to errors [10]. This has direct implication on the performance of quantum computing as it causes issues such as quantum decoherence, logical errors, superposition disruption, etc., directly affecting the computing accuracy. Besides, error correction systems also require substantial computing resources posing additional practical challenges [11]–[14]. Researchers are making efforts to address quantum errors through the development of quantum error correction codes and fault-tolerant quantum computing architectures. Various approaches such as surface code [15]–[17], topological qubits [18], and error-mitigation techniques [12] are being sought to simprove the resilience of quantum computers against errors.

In recent times, electric circuits have garnered significant interest for their capability to replicate complex physical behavior such as topological physics [19] (*i.e.,* topological edge states and corner modes) [20]. There have also been some efforts to emulate basic quantum behavior in electric circuits. It has been shown that a group of LC resonators can classically emulate the characteristics of all the universal quantum gates [21]. Based on this insight, in one of such recent works, a novel framework for emulating quantum circuits using LC oscillators has been introduced [22]. The dynamics of a set of LC oscillators are precisely tuned to accurately emulate the quantum gate processes within the circuit. The proposed framework can successfully emulate all the universal quantum gates required for universal quantum computation. In this manuscript, we propose an LC-oscillator-based quantum circuit emulator framework for simulating quantum error correction circuits. This framework is designed for the simulation of quantum error correction circuits, specifically addressing two prevalent quantum errors: 1) bit-flip error, and 2) phase-flip error. Other types of errors such as gate error, measurement error, etc. are kept as a further scope of research in future.

The organization of the paper is as follows. In section 2, we briefly discuss two types of quantum errors and their implications in quantum circuits. In section 3, we provide a brief overview of the LC resonator-based quantum gate simulation framework. In section 4, our proposed quantum error correction framework is elaborately discussed.

2. Bit Flip Error and Phase Flip Error

A bit flip error in quantum computing refers to the alteration of the state of a qubit from 0 to 1 or vice versa in any stage of the computation. Pauli X-gate is used to perform a bit flip operation.

$$|0\rangle \rightarrow |1\rangle$$

$$|1\rangle \rightarrow |0\rangle$$

$$\alpha|1\rangle + \beta|0\rangle \rightarrow \alpha|0\rangle + \beta|1\rangle$$

This error can arise due to environmental factors or imperfections in quantum gates, leading to the corruption of quantum information.

On the other hand, a phase flip error involves a change in the phase of a qubit's quantum state. Pauli-Z gate is used to perform a phase-flip operation.

$$|0\rangle \rightarrow |0\rangle$$

$$|1\rangle \rightarrow -|1\rangle$$

$$\alpha|0\rangle + \beta|1\rangle \rightarrow \alpha|0\rangle - \beta|1\rangle$$

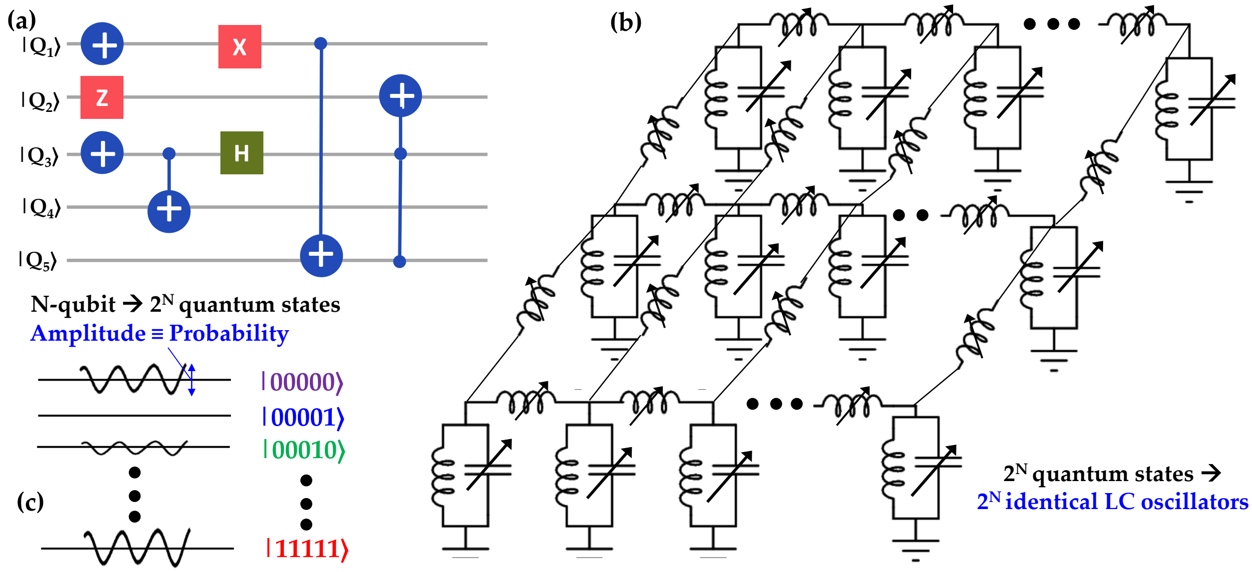

Figure 1: (a) Sample quantum circuit consisting of all the universal quantum gates. **(b)** A collection of LC oscillators representing an n-qubit quantum system. Each oscillator consists variable capacitance and coupling inductance. **(c)** The quantum states are represented by the amplitude and phase of the oscillations in the LC oscillators.

This alteration disrupts the superposition of quantum states, impacting the accuracy of quantum computations. Phase flip errors can occur independently or in conjunction with other types of errors, posing challenges for maintaining the integrity of quantum information. Bit-flip error and phse-flip error can also occur in conjunction, which can be represented by Pauli-Y matrix. Addressing and correcting these errors are essential components for ensuring the reliability and accuracy of quantum computations.

3. LC Oscillator-based Quantum Gate Simulation Framework

The quantum state of a qubit can be represented by the oscillation of an LC oscillator. The amplitude and relative phase of the oscillation can represent the amplitude and phase of the quantum states. For a system of N-qubits, 2^N identical LC oscillators are needed to completely represent the resultant quantum states. The oscillators can be inductively coupled to represent inter-dependency of the resultant quantum states. Motohiko Ezawa has shown that, by carefully imposing perturbation to the circuit elements, quantum gate processes can be perfectly mimicked [23]. The perturbation in the capacitance can be expressed as:

$$C(t) = \frac{C_0}{2}\left(\tanh\frac{t-t_1}{T} - \tanh\frac{t-t_2}{T}\right) \quad (1).$$

The perturbation in the coupling inductance governs the interplay between different resultant quantum states which can be expressed as:

$$L_C(t) = \frac{1}{2L_0}\left(\tanh\frac{t-t_1}{T} - \tanh\frac{t-t_2}{T}\right) \quad (2).$$

In both cases, the time t_1 and t_2 determine the perturbation period and the parameters L_0 and C_0 determine the perturbation amplitude. These parameters govern the oscillation dynamics and the interplay between the quantum states represented by the LC oscillators. Adopting this theoretical proposition, Islam *et al.*, has developed a SPICE-based scalable quantum circuit emulator framework for gate [22]-based quantum circuit simulation. Here, each gate process is performed by imposing precise perturbation in a time-multiplexed manner. Being a simulation framework, this framework is scalable to higher number of qubits and complex gate-based quantum circuit. We use this framework to simulate two types of quantum error correction circuits which will be discussed in the following section.

4 Quantum Error Correction by LC-resonator-based Classical Emulator Framework

Figure 2(a) depicts the schematics of a quantum encoder circuit. In this configuration, the qubit $|\psi\rangle$ is transmitted alongside two encoded ancilla qubits through a CNOT gate. $|\psi\rangle$ is encoded with two ancilla qubits performed by the CNOT gate. Instead of sending$|0\rangle$ or $|1\rangle$, the encoder sends $|000\rangle$ and $|111\rangle$ states to enhance robustness. A noise or error is introduced between the encoder and rest of the circuits. The encoded data is processed in the syndrome detection circuit to calculate the syndrome. If $|000\rangle$ or $|111\rangle$ is detected, then syndrome is calculated as $|00\rangle$ which translates to no bit-flip error. In other cases, different syndromes are detected as summarized in Fig. 2(b). These syndromes are utilized in the bit-error correction circuits to perform Toffoli gate operation in the flipped bit. If the probability of a single bit-flip is p, then, the probability of bit-flip error becomes $3p^2(1-p^2)$ which is less than p, if $p<0.5$. This way, the probability of an erroneous detection of flipped bit is reduced. Figure 2(c) displays the simulation waveform obtained from the quantum circuit simulation conducted in HSPICE. With a total of 5 qubits, inclusive of ancillas, the complete representation of the quantum system requires the utilization of 32 identical LC oscillators. The execution of CNOT or Toffoli gate operations is performed in the relevant oscillators in a time-multiplexed manner by introducing a precalculated perturbation in the coupling inductances as given in equation (2). The Toffoli gate is defined by the matrix,

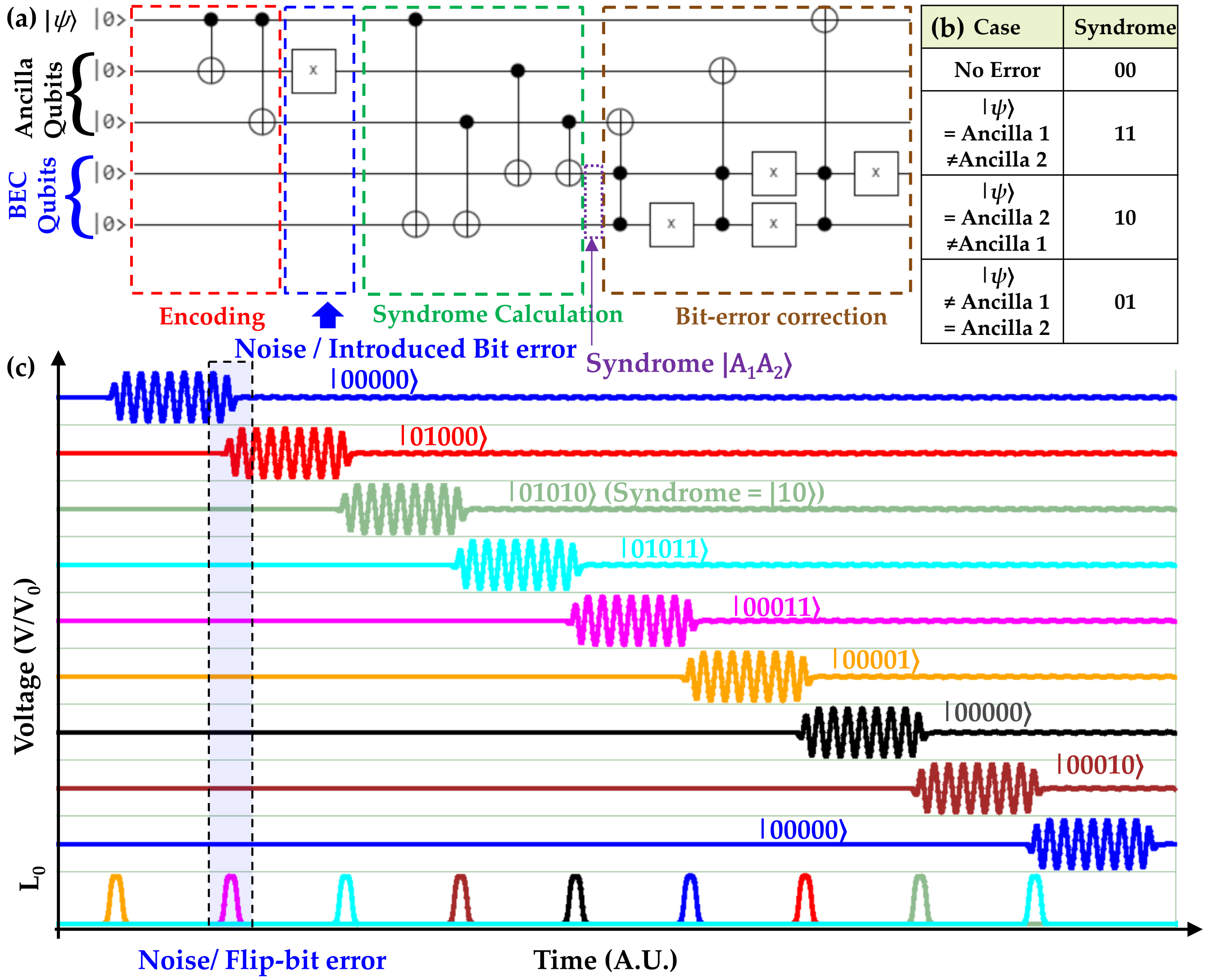

Figure 2: (a) Bit error correction circuit. **(b)** Syndromes calculated for each case of random bit error. **(c)** Waveform for the resulting quantum states at different stages for $|\psi\rangle = |0\rangle$.

$$U_{\text{Toffoli}} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

The Toffoli gate operation completely transfers the oscillation to the adjacent LC oscillator. After the BEC operation, the resultant quantum state becomes $|00010\rangle$ which indicates that our LC-oscillator-based framework has successfully performed the bit-error correction process and recovered the flipped ancilla bit by Toffoli operation keeping the other two bits intact.

Figure 3 depicts Shor's nine-bit error correction circuit, which tackles both bit-flip and phase-flip errors[24]. Here, the Hadamard gate is used to convert a phase-flip to a bit flip. The Hadamard gate is defined by the matrix,

$$U_H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

The gate matrix can be expressed as, $U_H = e^{-\frac{i\pi}{4}} U_{\frac{\pi}{2}} U_{MIX} U_{\frac{\pi}{2}}$. Hence, in our framework, we implement the Hadamard operation by sequentially performing a $\frac{\pi}{2}$-phase shift gate process, mixing gate process, then a second $\frac{\pi}{2}$-phase shift gate process followed by a $\frac{\pi}{4}$-phase shift process. As we can see in Fig. 3 at the preliminary stage of this circuit, phase flip correction takes place while the subsequent part of encoding section addresses three-qubit bit-flip code. Here, Hadamard gates allow correction of phase-flip errors by parity checks. It's noteworthy that this correction operates on a distinct basis, expressed as HZH = X, where the Z gate represents a phase flip error, and the X gate corresponds to an individual bit-flip. We simulate the circuit in our framework in a time

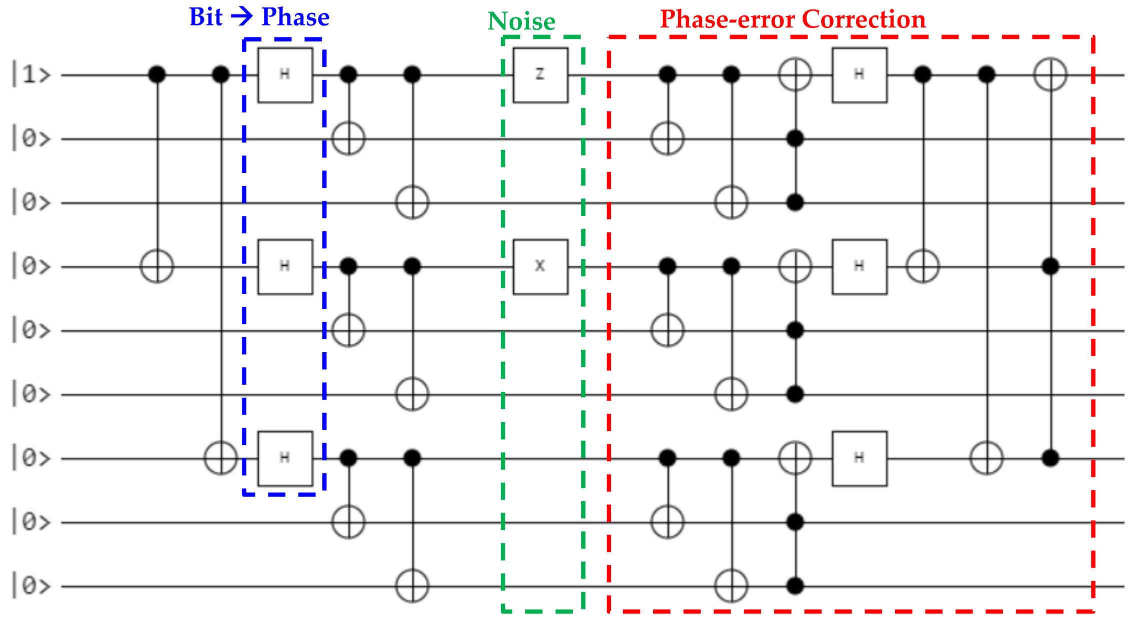

Figure 3: Phase error correction circuit with introduced phase error.

multiplexed manner as shown in Fig. 4. We perform a phase flip operation by introducing a Z-gate as shown in Fig. 3. The phase-flip is converted to a bit-flip and the phase is successfully retrieved as the final quantum state is $|100100100\rangle$. Our innovative framework successfully addresses both random bit-flip and random phase-flip errors.

5. Conclusion

We have developed a quantum circuit simulation framework for quantum error correction using SPICE. In this framework, a set of LC resonators accurately represents a system with any number of qubits. Within this framework, a collection of LC resonators serves as a precise representation of systems featuring varying qubit numbers. By fine-tuning the resonator capacitances and coupling inductors, we can effectively emulate universal gate processes. Utilizing the SPICE-based emulator framework, we can proficiently simulate two categories of quantum error correction circuits, successfully locating and rectifying both random bit-flip and phase-flip errors. What sets our emulator apart is its modular and scalable design, allowing seamless expansion for larger quantum systems with an increased number of qubits. Each quantum state is accurately portrayed through identical LC resonators, showcasing the versatility of our approach. Unlike hardware implementations, our simulation-driven methodology eliminates concerns tied to hardware complexity. This provides a straightforward controllability over inductor and capacitor values in the circuit simulation, contributing to the efficiency and adaptability of our quantum error correction framework.

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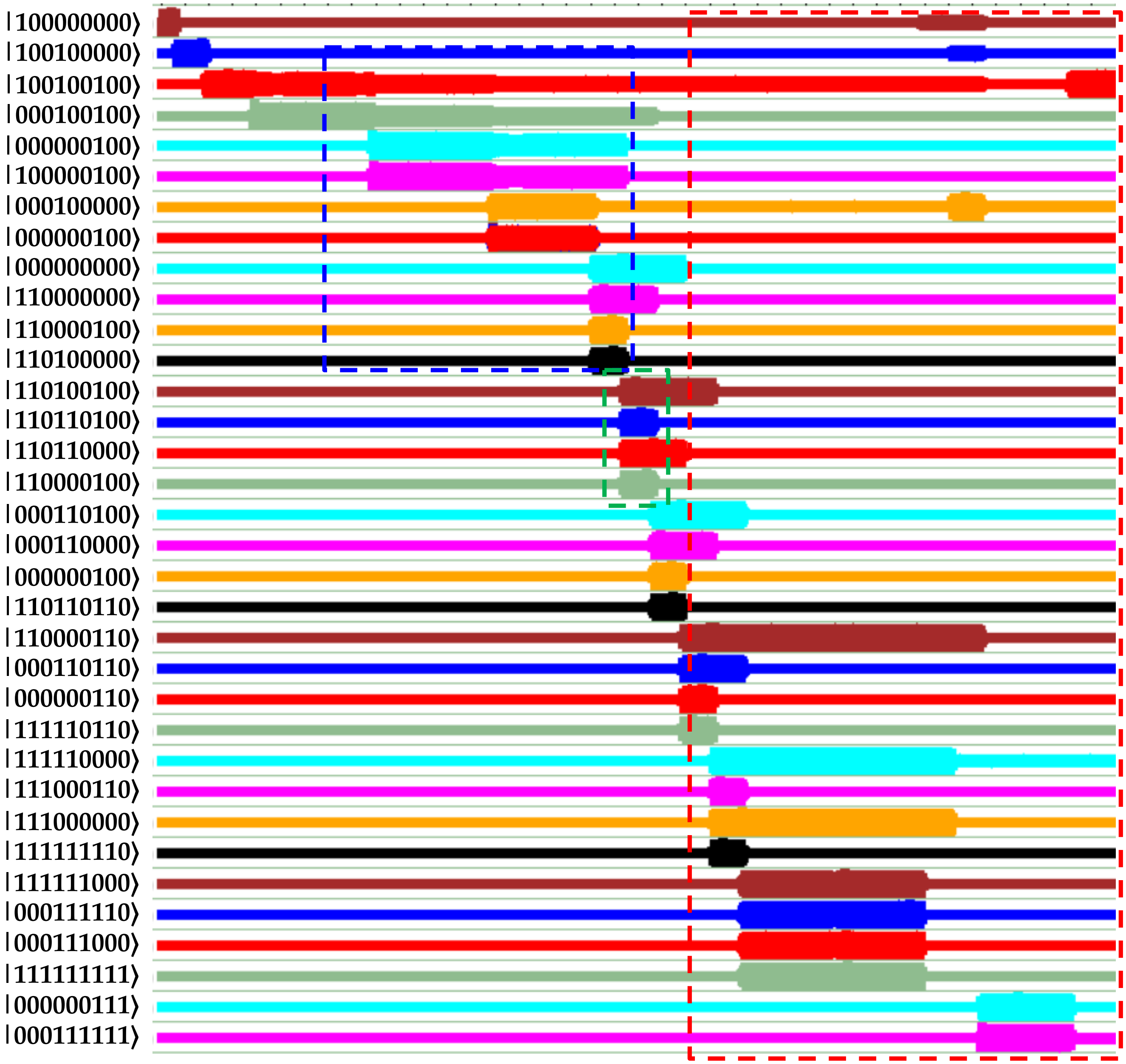

Figure 4: Simulated waveform for phase error correction circuit for an initial state $|\psi\rangle = |1\rangle$.

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